The Arithmetic Teacher

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Volume II Number 1

February 1955

Perspective in the Field of Arithmetic

B. R. Buckingham
Stuart, Florida

One of my colleagues at Ohio State University, although he taught courses in the field of elementary education, used to dismiss arithmetic with the remark—often repeated during the twenties—that the subject had come to a standstill, that there was little more to be learned about it, and that those who concerned themselves with it as a field of inquiry were dealing with trivialities. We knew all we needed to know, said he, about arithmetic and all of any consequence that we were ever likely to want to know.

I fancy too that my colleague, if he had spoken his full mind, would have said that arithmetic is a hard subject, an unlovely subject, and a subject altogether ungrateful, demanding the strength of the young and repaying with disappointment.

Although such an attitude was never really correct and although (so far as it involved a prophecy of stagnation) it has been brilliantly falsified, I can at least see how my professorial friend got that way. The period I am speaking of was the middle and late twenties. The measurement movement had laid a heavy hand upon arithmetic and we were all trying for "attempts" as a sort of substitute for "rights" (to use Courtis' unfortunate terms) and putting our faith in mechanics and yet more mechanics. No really notable book had been written since Thorndike's "The Psychology of Arithmetic," published early in 1922; and that book, with all its merits, affected arithmetic for the worse. The only arithmetic was computational arithmetic, whereas in reality computation is only part of arithmetic. And the only learning theory was the drill theory applied to computation whereas there are other applications for drill and other learning theories than that which the extended conception of drill encompasses.

So if my university friend was wrong—magnificently wrong as matters turned out—we can at least understand his state of mind as we look back upon the barren period during which he held forth.

Historical Perspective

An historical perspective concerning arithmetic compels us to note the extremely varied and sometimes even antagonistic meanings which have attached to the term. On the one hand, we have this uninviting concept of arithmetic which is associated with little children and with the manipulation of digits. On the other hand we have the concept of the greatest mathematician of the nineteenth century whose aphorism you may remember: "Mathematics is the queen of science and arithmetic the queen of mathematics." This famous remark of Gauss's quite evidently referred to a type of arithmetic which the schools, no matter how advanced they may be, seldom teach.

Taking quite a different point of view, we note that among the Greeks arithmetic

had nothing to do with reckoning or computation. The term for that discipline, if it may be so called, was "logistic." Arithmetic was a philosophical, a mystical, subject, especially among the Pythagoreans. In the Middle Ages arithmetic was one of the seven liberal arts, and again it had little to do with computation.

In the sixteenth century, especially under the influence of Italian business men, the modern idea of arithmetic took root. Leonardo had brought in the Hindu-Arabic system, a system which permitted operations to be carried out with the figures themselves and without the intermediary of an abacus. It is said that 300 arithmetics were published in Italy in the sixteenth century. Every algorism pertaining to the operations was tried, including the algorisms which we now teach to children and employ on the occasions when we unfortunately have to figure and haven't a machine to help us. We think today of four operations. For a time in the Middle Ages there was no agreement as to the number and character of these processes. Even in the sixteenth century six operations were usually recognized—the four with which we are now familiar, plus duplation, which was multiplying by 2, and mediation, which was dividing by 2. For some reason the figure 2 had its own special niche in the thinking of our ancestors and anything they did with it was different from what they did with 3 or any larger number. And 2 is different. . . .

For example, every number can be expressed as the sum of powers of 2. This is one of the many theorems of that arithmetic not taught in our schools to which Gauss referred as the queen of mathematics. This theorem indicating the peculiar relationship which exists between all numbers and the number 2 makes possible the type of multiplying in which duplation is the only operation; in other words, in which we never multiply by more than 2. This method is said to be still used by Russian peasants. Suppose, for example, that we wish to multiply 218 by 47. Here is the way one might do it

$$\begin{array}{rr}
218 & 1 \\
436 & 2 \\
872 & 4 \\
1744 & 8 \\
3488 & 16 \\
3488 & 16 \\
\hline
10246 & 47
\end{array}$$

What we owe in arithmetic to the formative period during which the present concepts, algorisms, and procedures were developed can only be apprehended by taking a much longer perspective of the field of arithmetic than is possible on this occasion. Perhaps it may suffice to say that only through the sifting process of the Middle Ages and the early modern period did any but professional mathematicians come to appreciate the need for care in placing our figures as we compute with them. This may be humorously illustrated by the member of a group of seven men who, after pursuing some sort of deal in which $28 had been gained, claimed $13 as his one-seventh share. . . .

$28 \div 7 = 13$ Check

$$\begin{array}{rr}
 & 13 \\
\hline
7) & 28 \\
 & 7 \\
\hline
 & 21 \\
 & 21 \\
\hline
\end{array}
\qquad
\begin{array}{r}
13 \\
\times\ 7 \\
\hline
21 \\
7 \\
\hline
28
\end{array}$$

Division: 7's in $2=0$. 7's in $8=1$.

Multiply, subtract, and bring down 2. 7's in $21=3$. Answer is 13.

Check: $7\times 3=21$. $7\times 1=7$. Then $21+7=28$. Answer checks.

Coming down nearer to our own day we may remark that just as the world has changed in the past century or two, so also has arithmetic. Conditions in the United States then were so different from conditions today that we cannot fully apprehend the life of our forebears. One hundred and fifty years ago the first census had just been taken. A book of etiquette was

forbidding young ladies to order oysters in a restaurant. New York State was giving up its claim to Vermont. And at Harvard College students were taking courses in arithmetic!

Arithmetic in the Schools

In fifty years arithmetic was no longer a customary part of the curriculum of Harvard College and had become an entrance requirement. Thus the subject started on that remarkable cycle of change that carried it farther and ever farther down through the secondary school to the primary grades of the elementary school.

One would have supposed that in the transit from a university subject to a secondary-school subject and then to a subject for children of ages 6 to 12 the subject would have been enormously simplified. Such, however, was not the case. The business tinge which the subject had taken in its Italian version stayed with it. Moreover as we came down into the early years of the present century it was clear that a great deal of the business reflected in our arithmetic curriculum no longer existed. Chain problems, allegation, equation of payments, partnership with time, the various rules of partial payments, true discount—these had long ago ceased to have application in business and survived in the arithmetic books for their supposed value as mental discipline.

Twenty-five years ago a powerful committee known as the Committee on the Economy of Time existed. This Committee made several reports in the teens of the present century and on the basis of its recommendations so far as arithmetic was concerned many of the obsolete and worthless topics were abolished. Not only were the chapters and sections just mentioned recommended for oblivion, but also certain other topics—topics not directly connected with business—were similarly treated. Among them were cube root, compound proportion, and the general methods of finding the greatest common divisor and the least common multiple. Even simple proportion was for a time banned from the courses of study. That was a pity too because proportionality is one of the great functional ideas which arithmetic, as a system of thinking, should recognize. It is almost the only method by which plans may be made on the basis of experience.

The work of the Committee on Economy of Time had to be done. The arithmetic course was simply impossible. We now think that the Committee erred in some respects—that it ought to have cut out some topics which it left in and that it ought to have left in a few topics (proportion, for example) which it threw overboard. It is a fact that the Committee was strongly influenced by its conception of business practice. I suspect that to this Committee on Economy of Time business was the sole, or almost the sole, basis of arithmetic. If a topic was good business practice, that was sufficient reason for leaving it in the arithmetic course. It is for this reason that we still have in the course, as taught to elementary-school children, such subjects as insurance, banking, and investment (stocks and bonds, particularly). These have social value as well as a modest amount of business value, but no child is going to make use of them for a long time to come; and he had much better be devoting himself for the present to other matters. One of the next things that I expect to see happen to the curriculum in arithmetic is the elimination from it of topics which have only a deferred value.

I have already alluded to the fact that during the early and middle twenties arithmetic, though tempered to the shorn lamb, through the omission of many topics, had become a barren, mechanical ordeal in computation with the drill theory as the principal method. According to the drill theory, a process would be broken up into as many distinguishable skills as possible. These skills would be practiced each in isolation, without any concern for its meaning—this with the hope and expecta-

tion that the pupil would put these separately presented skills together into an understandable whole.

Late in the twenties the revolt against this sort of thing began. It began, of all places, in the Twenty-Ninth Yearbook of the National Society for the Study of Education. Heaven knows that volume was sufficiently devoted to the drill theory, but the chapters by at least two of the contributors sounded the new note.

Another note was sounded in the same publication. Washburne rendered the first report of the Committee of Seven, recommending the postponement of the various topics of arithmetic until children had reached higher mental ages. In simple terms this meant that on the basis of what appeared to be competent research the facts of addition, subtraction, multiplication, and division, and the processes of long multiplication and long division and each of the processes with fractions and decimals—all were to be pushed up into higher grades.

A tremendous controversy arose over these recommendations. Most of it was leveled at the insufficient and unscientific basis for the Committee's findings. The basis *was* insufficient, and it was unscientific. The methodologists could make the Committee's method look ridiculous. Nevertheless the Committee had something, and when its chairman in recent reports assures us that the findings of the Committee have nothing to do with the level at which a topic in its rudimentary aspects may be *begun*, but only to the point at which the topic, in the elementary-school sense, may be regarded as *completed*—I say when this allowance is made, we are bound to admit that with or without benefit of scientific rigor the Committee of Seven has given the people what they want.

Indeed, the drift of thinking in regard to arithmetic had already set in along the lines laid down by the Committee of Seven. When I ran an elementary school in the second decade of the present century, and for another decade thereafter, the courses of study in most cities called for the automatic learning of the addition and subtraction facts by the end of the second grade and for similarly automatic learning of the multiplication and division facts by the end of the third grade. This proved to be an impossible task for any but the bright children. The sheer bulk of the job to be done—running, as Osburn pointed out in his Corrective Arithmetic, to 16 or 18 hundred number facts—compelled teachers in practice to postpone the completion of this broad factual basis for computational arithmetic, no matter what the course of study had to say about it.

It was the same with long division. As long ago as 1928 some few courses of study were beginning to put long division over into the fifth grade. For many years it had held an undisturbed place, so far as the printed word was concerned, as a fourth-grade subject. At present most courses of study with which I am familiar and all the newer textbooks introduce division by divisors of two or more places above the fourth grade.

Along with this has come the dropping out of short division as an introduction to the process. In most schools and according to most present courses of study all division is long division except when used with one-figure divisors as a short cut. In the business world most people use the long form for dividing by a one-place number in spite of the fact that they were not taught that way.

Recent History

Many teachers now in service have participated in the significant trends in teaching arithmetic during the past twenty years. Brownell under the title "The Revolution in Arithmetic" described these trends in the first issue of THE ARITHMETIC TEACHER (February, 1954) and gave the steps leading to better present practice. Two yearbooks of the National Council of Teachers of Mathematics, the Tenth

(1935) *The Teaching of Arithmetic* and the Sixteenth (1941) *Arithmetic in General Education* were strong influences toward reform in both methods of learning and in curriculum content. At this same time courses of study were beginning to feature the newer vision of arithmetic as a school subject. This is reflected, for example, in the New York State course of study *Mathematics for Elementary Schools* (1937) in which even the title rejected the older concept of arithmetic as "number work to be learned by drill procedures."

More recently the Fiftieth Yearbook (1951) *The Teaching of Arithmetic* of The National Society of the Study of Education and the Twenty-second Yearbook (1954) of The National Council of Teachers of Mathematics, *Emerging Practices in Mathematics Education* have brought the modern movement up to date. The modern movement in arithmetic is characterized by such key words as *discovery, meaning, understanding, significance, resourcefulness, seeing sense, concepts, relationships*, etc. The implication is that children shall learn arithmetic not as an abstract science of numbers but as an avenue of understanding the quantitative in modern society. Arithmetic shall be functional, and to be genuinely functional both the process and the role of arithmetic shall be understood. Functional arithmetic is not restricted to buying and selling, it serves also in the social and cultural as well as the industrial activities of the well educated citizen.

EDITOR'S NOTE: Dr. Buckingham is now living in retirement after many years as a writer, researcher, teacher, and editor. His article will suggest that it is worthwhile, occasionally, to take the "long view" of a subject such as arithmetic. He would agree that it is too early to assess the importance of the current movement. The next twenty years should see great progress in refining modes of learning and in adaptation thereof to different levels of ability, experience, and maturity. The immediate future should be concerned with restudy of the content of the curriculum with attention to selection and arrangement of content in relation to kinds of pupils and to the functions of arithmetic not only as a body of knowledge and mode of thinking that is useful in our current society but also as a structure that possesses internal unity and relationships. Goals of instruction should be viewed not only as immediate but also as extending. A modern method of teaching establishes the materials plus the understandings for educational "transfer."

BOOK REVIEW

Arithmetic and Curriculum Organization, Vincent J. Glennon and Students, Bureau of School Service, School of Education, Syracuse University, 1954. 140 pages, $2.00.

This is the third of a series of publications that have been developed in Professor Glennon's classes. This booklet aims to give the teacher an insight into the content and method of teaching arithmetic in the modern elementary school. It seeks to improve the arithmetic in the classroom. The beginning chapters by Dr. Glennon deal with discussions of theories of teaching arithmetic and modes of organizing learning. Chapters IV through XII were prepared by students. Each of these chapters lists objectives, approaches, learning experiences, and evaluation techniques of some activity or topic for one grade of the sequence grades kindergarten through eight. An appendix gives a comprehensive list of available films and other materials.

No doubt the students who compiled the lists of objectives learning experiences profited greatly thereby. This is the kind of project that groups of teachers in any school might well carry out. It is interesting to note that these particular teachers did not restrict themselves to the purely mathematical aspects of an activity; for example, in the topic of transportation one finds the objective "write creative stories."

LOUISE A. MACPHEE, BEN A. SUELTZ

An Experimental Approach to the Division Idea

Dan T. Dawson, *Stanford University*
Arden K. Ruddell, *San Jose State College*

There are but few serious questions today about the theoretical value of meaning theory in teaching arithmetic. A previous article in The Arithmetic Teacher[1] clearly states the case for *meaning* in arithmetic. However, previously reported research deals primarily with transfer of learning, retroactive inhibition, and drill. Relatively few leads have been provided for teachers interested in specifics such as, "What methods do I use in presenting division?" or "Just how does one use manipulative materials?"

It was the purpose of this study to compare the relative effectiveness of common textbook practices in the introduction of the division of whole numbers with an experimental procedure based on a subtractive approach and a greatly expanded use of visualization devices. The following questions were posed:

1. How will achievement results be affected if the subtractive concept is used to teach the division of whole numbers?[2,3]
2. Will achievement be adversely affected if practice through object manipulation and visualization of the process replaces much of the paper and pencil drill?
3. Will children learn the inter-relationships between multiplication, addition, and division if division is taught by the subtractive principle?

[1] Brownell, W. A., "The Revolution in Arithmetic," The Arithmetic Teacher, I (February 1954) p. 1–5.

[2] Buckingham, B. R., *Elementary Arithmetic*, Boston: Ginn and Co., 1947, p. 236.

"This operation (division) is performed by bunched subtraction, with a tally (called the quotient) of the number of subtractions."

[3] Nordahl, Marguerite, "Arithmetic as Concept Building," *The Mathematics Teacher*, XXXV (November 1942) p. 316–20.

"Division, as a special case of subtraction, consists in determining the number of times the divisor . . . can be taken from the dividend. . . . The quotient shows the number of times the subtraction has been made."

Experimental Design

This experiment was conducted in twelve fourth grade classes in the Palo Alto Unified School District, each taught by a different teacher. Six classes composed a control group and six classes composed an experimental group. All phases of the study were completed by 280 pupils—142 pupils in the control group and 138 pupils in the experimental group.

Equating the Groups

The experimental and control groups were equated on the basis of: (1) chronological age, (2) intelligence quotient, (3) achievement in division, (4) total number of pupils in each group, (5) socio-economic background, (6) class time devoted to arithmetic, and (7) teaching ability of the teacher.

Teachers selected to participate in this study were rated as excellent by the administrative staff. Each expressed his willingness to devote extra time to this study. There was one control class and one experimental class in each school. Only a few pupils who made extremely high scores on the pre-tests were eliminated from the study.

Instructional Procedures

Control Group. The teachers in the control group continued to teach as they had in the past, using the California State Arithmetic Series[4] as a guide. They were encouraged to use any form of presentation, practice, or review they normally used in teaching arithmetic. None of the teachers in the control group had previously practiced any of the procedures

[4] Brueckner, L. T., Grossnickle, F. E. and Merton E. C. *Arithmetic We Use*. Grade 4. Sacramento: California State Department of Education. 1948.

specifically planned for use in the experimental group. In general, the teachers in this group began each class period with a brief oral discussion about "how" the process worked. Most of the class time was devoted to paper and pencil type practice.

Experimental Group. The teachers of the experimental group were asked to use a teaching procedure based upon the theory that children learn quantitative concepts more efficiently if they focus on arithmetic principles and generalizations. In particular, meaningful emphasis was placed on the presentation of representative objects and on the visualization of socially significant mathematical situations. The concept of division as a special case of subtraction was carefully developed and served as a major basis for the entire teaching procedure. Counting discs, spool boards, and place value charts were provided for each teacher. A major portion of each class period was spent in discussion, and the devices described were used extensively in visualizing the concepts presented. Practice periods were included as part of the lesson; then the children were asked to work six or eight written division examples. It should be noted, however, that typically children worked each example by three methods, with counters, with a spool board and with paper and pencil.

Evaluation of Instruction

Three diagnostic division tests were constructed to serve as pre- and post-test measures of achievement and retention. The experiment began in October and continued for twenty-two teaching days. No multiplication or division was taught or discussed in class during the ensuing seven weeks. A test for retention was then administered.

A structured interview was held with each pupil before and after the instructional period to collect data relevant to children's thought processes as they solved division examples.

The Findings

Tables I and II contain the test findings.

Test I was administered before the experiment began and again immediately following the formal instructional phase

TABLE I

SCORES ON TEST I
(FOURTH GRADERS TAUGHT DIVISION BY TWO DIFFERENT METHODS)

	Control Group $N=142$		Experimental Group $N=138$	
	Mean	S.D.	Mean	S.D.
Test I (Pre)	12.15	9.63	12.33	10.62
Test I (Post)	26.16	11.73	32.24	9.27

TABLE II

SCORES ON TEST II
(FOURTH GRADERS TAUGHT DIVISION BY TWO DIFFERENT METHODS)

	Control Group $N=142$		Experimental Group $N=138$	
	Mean	S.D.	Mean	S.D.
Pre-Test II-1	12.31	7.16	13.78	6.83
Post-Test II-1	10.16	6.95	14.00	6.95
Pre-Test II-2	3.82	5.25	8.62	6.03
Post Test II-2	4.25	5.55	7.97	5.95

of the study. Both of the groups made significant gains. A highly significant difference in favor of the experimental group was found between the gains of the two groups.

Pre-Test II (Parts 1 & 2) was administered as an achievement test immediately following the formal instruction period. Post-Test II (Parts 1 & 2) was administered seven weeks later as a test for retention. Part 1 of each test included all levels of difficulty presented during the instructional period. Part 2 of each test consisted of types of division which had not been discussed previously in class.

The findings of this study, as presented in Tables I and II, provide impressive evidence in answer to the questions posed.

1. *How will achievement results be af-*

fected if the subtractive concept is used to teach the division of whole numbers? The children in the experimental group scored significantly higher than the control group on the final post-achievement test. Likewise, on the measures of retention, the children in the experimental groups scored higher and showed a positive change while the control scores dropped significantly. The experimental group was better able to work examples not previously encounttered or discussed.

2. *Will achievement be adversely affected if practice through object manipulation and visualization of the process replaces much of the paper and pencil drill?* The data in Table I and II may be interpreted to advocate a teaching procedure which utilized manipulation of representative materials. Higher achievement, greater retention, and increased ability to solve examples in a new situation were found in the experimental group which devoted time to developing meanings, principles, and generalizations through the use of manipulative materials and visualization procedures.

3. *Will children learn the inter-relationships between multiplication, addition, and division if division is taught by the subtractive principle?* Types of understanding of division possessed by children in the third and fourth grades were identified through a pilot study-interview. This preliminary study revealed the following types of understanding of division:

(R) Rote Memorization. No understanding. The child responded with the correct answer quickly but was unable to tell why he knew the answer or how to find the answer.
(+) Additive. The child could prove the answer by a series of equal additions.
(−) Subtractive. The child could prove the answer by a series of equal subtractions.
(×) Multiplication. The child responded that the quotient multiplied by the divisor equals the dividend.
(T) Tables. The child repeated the appropriate multiplication table to himself, counting on his fingers to the correct response.
(I) Indeterminate. The child failed completely to recognize the algorism.

Mixed responses, combinations of those listed above, were observed in the interview conducted at the conclusion of the instructional period.

During this experiment the understanding of division, as expressed by a great majority of the children, was changed. The nature and kind of response changes have been categorized by the type of understanding expressed in the post-interview (Table III).

TABLE III

CHANGED RESPONSES IN TYPES OF UNDERSTANDING OF DIVISION

	(−)	(+)	(X)	(*R*)
Control Group	0	39	52	25
Experimental Group	63	49	59	6

In this study children who were taught the division of whole numbers through the subtractive concept utilizing representative objects developed a greater understanding of division and its inter-relationships with subtraction, multiplication, and addition than children who were taught by common textbook practices. Furthermore, fewer children, thus taught, relied upon rote memorization. The data in Table III provide striking evidence of this fact.

Recommendations for Teaching

How effectively can the division of whole numbers be taught by a method which relies heavily upon:

(1) the manipulation of representative materials for
(2) the visualization of the number concept, and upon
(3) subtraction as a basic principle in division?

According to the findings of this study,

the subtractive concept appears to be a fruitful procedure upon which to base the introduction of division of whole numbers. Therefore, it would appear that teachers in-service and teachers in-training should become thoroughly familiar with the concept of division as a special case of subtraction.

Three additional major recommendations are in order.

First, of paramount consideration in developing the inter-relationships in division is the understanding of the decimal system of notation and the concept of positional value. These concepts are basic to an understanding of the division process. It is apparent, then, that teachers need to give careful consideration to the systematic development of these ideas by children in the primary grades. Children need to be thoroughly familiar with these concepts before the division of whole numbers is introduced. In this study particular attention was given to the development of the basic ideas relevant to the decimal system of notation.

Second, teachers need to understand and use a wide variety of manipulative devices in teaching arithmetic. It is of critical importance that children should be encouraged to manipulate representative materials and to discover mathematical principles and concepts through visualization and manipulation throughout this development. Much class time can profitably be devoted to discussion.

Third, it is well established that specific short practice periods help fix skills. In this study practice or drill included the use of visualization materials as well as paper and pencil work. From the results of this study, this would appear to be a profitable approach to practice procedures.

Summary

Groups of fourth grade children were used to study the relative effectiveness of commonly described practices in the introductory teaching of the division process. Higher achievement, greater retention, and increased ability to solve examples in a new situation were found in the experimental group where increased time was devoted to developing meanings, principles, and generalizations through the use of manipulative materials and visualization procedures.

EDITOR'S NOTE: This experimental study gives strong evidence in favor of using visual-manipulative devices and materials for the beginning stages of learning division. Some research workers will want to study the original data before attaching reliability to the usual tests of significance of difference of means when applied to these data because of the shapes of the distributions. Note that in some cases the standard deviation exceeds the mean.

We need many more studies of this kind and particularly studies carried over a longer period of time. Teachers who wish to know more about the experimental procedures used in teaching and the tests used in measuring in this study may write to Dr. Dawson at Stanford University.

BOOK REVIEW

Arithmetic—Children Use It!, Edwina Deans, Association for Childhood Education, 1200 Fifteenth St. N. W., Washington, D.C., 1954. 56 pages, paper, $0.75.

A readable bulletin compiled by Dr. Deans who is the elementary school supervisor, Arlington County, Virginia. The basic materials assembled into this book were supplied from classroom and home situations from various sections of the country. Dr. Deans selected and welded the anecdotes into chapters showing typical situations for children ranging from age four through eleven. The basic concept held throughout is that arithmetic ideas should be met in functional settings.

The bulletin is valuable for teachers and parents who want to help pupils learn functional arithmetic in a way in which they will appreciate its usefulness and understand what it is and how numbers should be used.

BEN A. SUELTZ

Zero's Little Blessing

Elizabeth Ann Bass
La Mesa, California

(*The scene takes place in the world of numberland where we find Zero in a very unhappy mood.*)

Zero: Oh, boo hoo, I'm so unhappy!

(EIGHT enters, looking concerned)

Eight: Did I hear someone crying in here? Why, Zero, whatever is the matter?

Zero: Oh, Eight, the most terrible thing happened. You know how I've always wanted a child!

Eight: Why, yes, but—

Zero: Well, lately I've been praying extra hard for a child and—sniff—look what I got!

(DECIMAL comes out from behind ZERO'S chair)

Eight: Oh, my goodness!

Zero: (Looks at Decimal and cries louder). Oh, what have I done to deserve this?

Decimal: My name is Decimal Point.

Eight: What's a Decimal Point?

Decimal: I'm a Decimal Point.

Eight: But what do you do?

Decimal: I—I don't know.

Zero: Sniff! (blows nose)

(THREE enters, running)

Three: I bet I can run faster than—(Looks and points at Decimal) What's that?

Zero: Oh, boo hoo!

Eight: Now you've done it! That's Zero's child.

Five: Hey, you know he kinda looks like a little Zero only he doesn't have a hole in the middle.

Three: Oh, silly! He is the hole in the middle! He just doesn't have an outside.

Zero: When those pygmy fractions find out about this they'll really ridicule us!

Five: No, they won't! I've got a wonderful idea. We can use Decimal instead of a fraction to represent something smaller than one.

Decimal: But how can I do that?

Characters in Zero's Little Blessing

Five: It's very easy. Eight and Zero show Eighty. (Eight and Zero stand up to form 80.) Now, Decimal, you stand on the other side of Zero. Now, Eight and Zero, you are still eighty because you are on the left side of Decimal.

Decimal: Then what good am I?

Five: Just be patient, I'll show you. Three, stand next to Decimal.

Three: O.K., but I don't see why. (Stands next to Decimal)

Five: Now, instead of being eight hundred three you are eighty and three-tenths.

Eight: But why?

Five: Because Decimal separates whole numbers and fractions. The first number to the right of Decimal represents tenths, the second one represents hundredths, the third represents thousandths, and so on. Now, I'll stand next to three and we'll be eighty and thirty-five hundredths.

Decimal: Do you mean I'm *really important?*

Five: I'll say! We'll be a lot more useful from now on.

Three: I look the same but see how my value has changed!

Eight: Well, Zero, what do you think of your child now?

Zero: I'm sorry about what I said, Decimal. You turned out to be a blessing in disguise.

EDITOR'S NOTE: Elizabeth Ann Bass wrote "Zero's Little Blessing" when she was eleven years old and in the sixth grade at Lemon Avenue Elementary School in La Mesa, California. Her purpose was to capture the interest of her classmates in the importance of zero and the decimal point. The play could easily be extended to include more phases of understanding of the role of zero and of the decimal notation. Large cut-outs of numbers are fastened to the clothing of the actors.

Someone may wish to extend the play into another scene to include such functions of zero as representing "no value" as a zero score, as a "place holder" in the formation of numbers, and as a "point of origin" such as zero in a temperature scale. These concepts are within the experience of sixth graders.

ADDING A COLUMN OF FIGURES

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8
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6
—

We would like our pupils in intermediate grades to be able to add accurately and without hesitation. Those who add the column at the left beginning at the bottom should be able to proceed easily, saying to themselves only the partial sums as 11, 14, 16, 24, 28, 35. What about seeking out special combinations such as $7+8=15$, $6+4=10$, $5+3+2=10$ and then adding $15+10+10=35$? Many intelligent adults do this and do it very well. Should we encourage such an intermediate procedure among pupils in grades three and four? Should we be content with children in intermediate grades getting correct answers regardless of their procedure even though it requires four times as long as by the direct process? Would we permit certain pupils to count dots or count on their fingers even in grade five?

In general, most schools want, as an end product, the ability to add directly and with dispatch. They may use a number of different procedures for discovering and establishing the basic number combinations but as a final technique they want accurate and reasonably rapid methods of addition. If, however, some pupils can do better by indirect methods, it seems advisable to permit such devices as "grouping combinations" and other intermediate steps. Accuracy is more important than speed. Both speed and accuracy can be achieved if teachers work toward that goal. We must remember that Susie's normal rate of speed differs from that of Sam.

The Semantics and Grammar of Arithmetic Language

(Or, What Is "Meaning" in Arithmetic?)

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Ask a woman to close her eyes and tell you what she visualizes when you say the word *handsome*. Ask a man to do the same and tell you what he visualizes when you say the word *pretty*. Now ask both to close their eyes and tell what they visualize when you say *two hundred eighty-six*.

If the people you ask respond as have the hundreds of teachers the writer has questioned, the responses will usually be, "a man," "a woman," and "the written number *286*." Very seldom does a person visualize the *written symbols* for the words *handsome* and *pretty*. Neither does he visualize a collection of about 300 *things* for the words *two hundred eighty-six*.

The word *handsome* is an adjective when it relates to persons, places, or things. The same word, however, can also be a noun. It is a noun when it is being considered alone as a word-symbol. For example: "The word *handsome* has eight letters and two syllables. It is composed of the words *hand* and *some*. Its dictionary meaning is— — —."

The grammarian who spends much of his time studying and analyzing words might visualize the word-symbol for *handsome* when hearing the spoken word-symbol for *handsome*. The rest of us, however, do not ordinarily visualize the word-symbol itself, but rather what the symbol represents. In our lives the meaning and importance of such a word as *handsome* reside not in the nature of the word itself but in the person, place, or thing, which the word modifies. Such words as *handsome* and *pretty* have more meaning to us when they are adjectives than when they are nouns. They have more meaning for us non-grammarians when the word-symbols call forth a visualization of what the symbols represent than when they call forth a visualization of the symbolism itself.

A number-symbol, such as *286*, also can be an adjective or a noun. It is an adjective when it refers to persons, places, or things. It is a noun when the number is being considered as a symbol. (A number is a noun also when it is a house number, telephone number, registration number, etc.)

"There are 286 children enrolled in the school." *286* in this sentence is an adjective.

"The number *286* is 1 more than 285 and 1 less than 287; it is composed of 6 units, 8 tens, and 2 hundreds; it is an even number; its factors are etc., etc." *286* in this sentence is a noun.

The mathematician who spends much of his time studying and analyzing numbers as symbols—their nature and relationship with other numbers—might understandably visualize the *written* number-symbol when hearing the spoken word-symbol for *two hundred eighty-six*. The rest of us, however, should not ordinarily visualize the number-symbol itself, but rather what the symbol represents, since in our lives the meaning and importance of such a number as *286* reside not in the nature of the number-symbol itself, but in the persons, places, and things which the number modifies. In other words, such numbers as 8, 15, 286, 423.07, $7\frac{3}{4}$ have more meaning to us when they are adjectives than when they are nouns. They would be of greater value to us non-mathematicians, therefore, if the number-symbols called forth a visualization of what the symbols represented rather than the symbolism itself.

When an adjective is infrequently en-

countered or has not been encountered at all in a concrete situation, that adjective can not very well call to mind a visualization except that of the appearance of the word-symbol itself.

Similarly, when a number is infrequently encountered or has not been encountered at all in a concrete situation that number can not very well call to mind a visualization except that of the appearance of the number-symbol itself.

It would seem, therefore, that in school we should direct children's attention more to the quantitative nature of the persons, places, and things that numbers represent than to the nature of the number symbolism itself. Especially is this true when computation is involved. For seldom, if ever, in life outside of the classroom does one add, subtract, multiply, or divide numbers that are nouns—telephone numbers, box-car numbers—or compute with numbers just for the sake of computing with numbers.

Semantics is the study of the meanings of words; i.e., what word-symbols represent. Grammar is the study of the nature of words (etymology) and the relationships of words in sentences (syntax); i.e., the *structure* of language expression.

Applied to arithmetical language, the word semantics could refer to the meanings of numbers; i.e., what arithmetic-symbols represent. Grammar, as applied to arithmetical language, could refer to the nature and structure of the number system and the relationships of numbers within the system.

Briefly, semantics can be thought of as referring to the relation between symbolism and that which the symbolism represents; grammar can be thought of as referring to the nature and structure of the system of symbolism itself.

"Meaning" in Arithmetic

In recent years much emphasis has been placed upon the importance of developing *meaning* in arithmetic. Many authorities, however, have interpreted meaning in arithmetic to be *mathematical* meaning.[1] If the analogies between *word*-language and *arithmetic*-language as discussed here have any validity, perhaps the meaning of meaning in arithmetic should be given further consideration.

The learning by young children of word-language starts with vocabulary building, i.e., with associating words heard with the things, actions, and relationships which are in the child's immediate environment. *Words and ways to put words together in sentences are first learned by listening to others talk and by talking oneself—not by studying grammar.* Grammatical rules concerning parts of speech, tenses, cases, number, modifiers, subjects, predicates, phrases, clauses, etc., are studied only *after* the language has been learned through imitation and used as a means of communication for a number of years. In other words, in the growth and development of the child the *semantics* of word-language precedes and is basic to the conscious study of the *grammar* of word-language. (It is pretty well agreed that a foreign language is best learned in this way also.)

Applying the same kind of learning procedure to arithmetic-language, children should learn to express quantities and quantitative relationships with arithmetical symbols *before* they study the intricacies of the number system itself.

The mouthing of words and the knowing of rules of etymology and syntax do not in themselves require or guarantee a knowledge of the *meanings* of the words and sentences uttered.

Likewise, the computing with numbers and the knowing of rules and principles of place value and computational procedures do not in themselves require or

[1] "Instruction in arithmetic aims at developing meaning and understanding and in helping the child see sense in what he does. This means teaching arithmetic with attention to the number system, for meaning in arithmetic inheres in the system itself." Charlotte Junge, "The Arithmetic Curriculum—1954." THE ARITHMETIC TEACHER, Vol. I, No. 2, April 1954.

guarantee a knowledge of the *meanings* of the numbers and algorisms manipulated.

Regardless of the method used to obtain the answer to any of the following algorisms (and there are *several* correct ways of obtaining an answer) or regardless of whether the computor does or does not understand *why* his method of manipulating the numbers works, he does have an understanding of the *meaning* of the algorisms if he can visualize some such situations as the following:

(a) For $\begin{array}{r} 4 \\ +\underline{3} \end{array}$

3 books are put on a pile of 4 books making a pile of how many?

(b) For $\begin{array}{r} 42 \\ -\underline{28} \end{array}$

42 chairs are needed altogether. With 28 on hand how many more must be obtained?

(c) For $\begin{array}{r} 15 \\ \times\underline{34} \end{array}$

Each of 34 children has 15 tickets to sell. How many tickets do they have altogether?____

(d) For $32\overline{)254}$

A collection of 254 stamps is divided as evenly as possible among 32 children. How many does each child receive?

(e) For $3\frac{1}{2} \div 12\frac{3}{4}$

A pole $12\frac{3}{4}$ feet long is set in a hole $3\frac{1}{2}$ feet deep. What part of it is in the hole?

(f) For $\begin{array}{r} 7.9 \\ \times\underline{3.2} \end{array}$

The property being 7.9 miles long and 3.2 miles wide contains how many square miles? _____

(g) For $4.6\overline{)3497}$

About how many families are in a town of 3497 population assuming the average number of people in a family is 4.6?

A child's ability to visualize and describe a concrete situation represented by an algorism comes from frequent opportunities to express with algorisms his own experiences in similar concrete situations.

Also, a child is helped in learning to write and read algorisms with meaning if he verbalizes the arithmetic symbolism with language he understands, i.e., with language that calls to mind visualizations of things.

What does one visualize when he hears spoken, "Four plus three equals seven"? Of course, only the appearance of the arithmetical symbolism itself, $4+3=7$ or $\begin{array}{r} 4 \\ +\underline{3} \\ 7 \end{array}$. It is easier to visualize that which the symbolism represents if one hears spoken, "Four and three are seven." Similarly, for "Seven minus three equals four," the only thing that can be visualized is the symbolism, $7-3=4$ or $\begin{array}{r} 7 \\ -\underline{3} \\ 4 \end{array}$. However, "Seven take away three leaves four" can call to mind something happening to things.

The terms *plus, minus, times, multiplied by, divided by,* and *equals* are technical mathematical terms and call to mind only the symbols $+ - \times \div \overline{)\quad} =$. There is grave danger that these technical terms will be considered nonsense syllables if they are introduced to children *before* the meanings of the operational signs are well established. The operational signs show what is happening to amounts, sizes, capacities, etc., of *things* and should be read and thought of accordingly.

$2\times 8=$ could be read, "Two eights are how many."

$12\div 3=$ could be read, "How many three's are in twelve?" or "twelve divided into three equal parts are how many?"

$\frac{2}{3}\times 12=$ could be read, "Two thirds of twelve are how many?"

$12\times\frac{2}{3}=$ could be read, "Two thirds taken twelve times are how many?"

$12\frac{3}{4}\div 2\frac{7}{8}=$ could be read, "Two and seven eighths are contained in twelve and three fourths how many times?"

$2\frac{7}{8}\div 12\frac{3}{4}=$ could be read, "What part of twelve and three fourths is contained in two and seven eighths?"

$8.7-3.9=$ could be read, "eight and seven-tenths take away three and nine-tenths are how many?"

$.81\times 3.43=$ could be read, "eighty-one hundredths of three and forty-three hundredths are how many?"

$3.43\times .81=$ could be read, "eighty-one hundredths taken three and forty-three times are how many?"

If an algorism is thought of as a shorthand way of writing a sentence that has adjectives (numbers), nouns understood, and verbs either expressed or understood, then the learner can be helped to visualize things and what is happening to them. For example, the addition symbolism could be introduced in the following manner:

Suppose the arrangement of objects is thus: □ □ □ □ □ □ □

1. "Combining four blocks and three blocks make seven blocks."
2. "4 blocks and 3 blocks are 7 blocks." (Drop *combining* and substitute the numerals for the number words and *are* for *make*.)
3. "4 blocks+3 blocks=7 blocks." (Substitute + for *and* and = for *are*.)
4. "4+3=7" (Verbalized as 4 and 3 are 7.) (Drop the word *blocks*, for 4 of anything and 3 of the same thing are 7 altogether.)

The subtraction, multiplication, and division symbolisms can be introduced as shorthand expressions in similar fashion.[2]

If meaning in arithmetic inheres in the semantics of arithmetic-language rather than in its grammar, perhaps the nature of the number system has been over-emphasized in teaching young children arithmetic. "Helping the child see sense in what he does" when he manipulates numbers in computation is a very laudable objective. This can be accomplished, however, without a head-on attack upon units, tens, hundreds, place-holders, etc.

Experiment and Discovery

Since answers can be obtained in different ways, allow the child to experiment and find the computational process best suited for him at his present stage of development. As he experiments with different ways to manipulate numbers he can be led to the discovery of certain rules and generalizations that are within the range of his comprehension.

[2] See pp. 104, 119 and 140 of *Guiding Children's Arithmetic Experiences*, by J. Allen Hickerson. Prentice-Hall, Inc., New York. 1952.

Computation is usually the process of separating numbers into parts, performing the indicated operation with the parts, and putting together the partial answers. Since numbers can be separated into parts in many different ways and the parts put together in different orders, children can experiment to find these various ways and orders.

Suppose 19 and 33 are being combined.
19 can be separated into 10 and 9.
33 can be separated into 30 and 3.

The following, then, are a few ways of adding the parts and putting the subtotals together:

(a)

$$\begin{array}{r}19\\+30\\\hline 49\end{array}\qquad\begin{array}{r}49\\+\ 3\\\hline 52\end{array}$$

(b)

$$\begin{array}{r}10\\+33\\\hline 43\end{array}\qquad\begin{array}{r}43\\+\ 9\\\hline 52\end{array}$$

(c)

$$\begin{array}{r}10\\+30\\\hline 40\end{array}\qquad\begin{array}{r}9\\+\ 3\\\hline 12\end{array}\qquad\begin{array}{r}40\\+12\\\hline 52\end{array}$$

(d)

$$\begin{array}{r}9\\+\ 3\\\hline 12\end{array}\qquad\begin{array}{r}10\\+30\\\hline 40\end{array}\qquad\begin{array}{r}12\\+40\\\hline 52\end{array}$$

(e)

$$\begin{array}{r}10\\+30\\\hline 40\end{array}\qquad\begin{array}{r}9\\+\ 3\\\hline 12\end{array}\qquad\begin{array}{r}40\\+10\\\hline 50\end{array}\qquad\begin{array}{r}50\\+\ 2\\\hline 52\end{array}$$

Help the children discover for themselves that the "carrying" technique is a short-cut way of performing (d). They can do this *without* being taught "units," "tens," "hundreds," etc.

Take another example: $\begin{array}{r}72\\\times 38\\\hline\end{array}$. 72 can be separated into 70 and 2, and 38 into 30 and 8. Multiply the parts.

$$\begin{array}{r}70\\\times 30\\\hline 2100\end{array}\qquad\begin{array}{r}2\\\times 30\\\hline 60\end{array}\qquad\begin{array}{r}70\\\times\ \ 8\\\hline 560\end{array}\qquad\begin{array}{r}2\\\times 8\\\hline 16\end{array}$$

Put the partial products together.

$$\begin{array}{r} 2100 \\ 60 \\ 560 \\ 16 \\ \hline 2736 \end{array}$$

The order of multiplying the parts can be reversed.

$$\begin{array}{r} 72 \\ \times 38 \\ \hline 16 \\ 560 \\ 60 \\ 2100 \\ \hline 2736 \end{array}$$

The partial products 16 and 560 can be added mentally. Likewise, 60 and 2100.

$$\begin{array}{r} 72 \\ \times 38 \\ \hline 576 \\ 2160 \\ \hline 2736 \end{array}$$

Now, the child can see why the 0 in 2160 can be omitted if the computor wishes. However, there really is no reason for omitting the zero since the partial products are 576 and 2160, not 576 and 216.

Tricks of "carrying," "borrowing," "subtracting and bringing down," "indenting," "canceling," "inverting and multiplying," "counting off decimal places," "moving" decimal points, etc., can be discovered and invented by children if they are not "taught" the rules and principles but are allowed to *discover* and *formulate* them for themselves.

The best way to gain an understanding of the rules and principles of the grammar of word-language is by discovering and formulating them oneself based on analysis of usage. The poorest way to learn the rules of grammar is to be *given* them and then being told to apply them.

Likewise, the best way to gain an understanding of the grammar of number-language is by *discovering* relationships and by formulating rules and principles based on analysis of usage. The poorest way to learn is to be *given* the rules and principles governing the number system and being told to apply them.

Children can gain a much better understanding of arithmetic symbolism if they are taught inductively rather than deductively.

The major emphasis of the arithmetic program in the elementary school should be to help children solve their everyday problems by having them (a) learn to represent with arithmetical symbolism quantities and quantitative relationships and (b) learn to compute with meaning and efficiency. Attention to the nature of the number system should be of secondary importance and be contributory to the major purpose.

EDITOR'S NOTE: Professor Hickerson calls attention to the need for association of numbers with concrete quantities because children develop concepts through this association. However it is probably wise practice to work gradually toward the "mathematical" concept of a number so that children may proceed with operations not encumbered with concrete materials. The artistry in teaching comes at least in part through knowing when to and how to guide pupils into more mature levels of thinking and performance. Mr. Hickerson believes in the inductive approach with provision for "discovery" by pupils. How do you like his "semantic" argument?

EXCEPT IN THE CASE of leap years, any holiday such as the Fourth of July falls one day later in the week this year than it did last year. Can you explain this by simple division?

Concept of Money via Experience

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The difficulty which children encounter in the reading and interpretation of problems is a concern of every teacher of arithmetic. This has come about through the functioning of several factors, including the following:

1. The vocabulary control applied to most arithmetic books is not comparable to that used for reading textbooks of the same grade level.

2. Frequently, the variety of new concepts introduced in a single set of problems is baffling. To stop and develop each concept would leave little time for computation. Hence the teacher is faced with what seems a necessity for making meager explanations and proceeding to the problem solving.

3. There is little correlation between arithmetic and other curriculum subjects. Hence development of needed concepts is confined largely to the arithmetic period.

4. Definite instruction in the reading skills needed for interpretation of arithmetic textbooks is seldom given, as such. This may be due, largely, to the time element involved.

5. Detailed suggestions for the development of concepts needed in the use of reading textbooks are provided in teachers' manuals. They have served to make teachers conscious of the need of a background of experience, to take to the reading of stories. A similar experience background is not usually provided for the study of problems. There has not been sufficient carry-over from reading methods to attain this end.

6. While children are grouped in reading and provided with material suited to their various instructional levels, no such differentiation is made when the reading and the interpretation of problems are encountered.

One phase of arithmetic which poses a vocabulary problem is that dealing with money. *How much money?* appears a simple question at first thought. However, it may be confusing to the child whose thought and experience has been with the concept *How many.* Furthermore, many a child may be able to compute an amount correctly without having even a vague concept of its meaning or value in reality.

The experiences of a fourth grade class, as described in this article, grew out of a felt need for a wider experience with, and a better understanding of, the common currency which is used in our every day transactions. Each activity engaged in suggested possibilities for wider experiences. When the school opportunities seemed to be exhausted, the door was opened by a leader in our community for a culminating excursion into an industrial area beyond. The natural motivation for the initial activity, and the development of the succeeding experiences are herein described.

The Motivating Incident

The fourth grade planned a breakfast at the local hotel. A duty of the finance committee was to collect the fees for breakfast, tips, and the expenses of a guest. They found themselves the custodians of the amount of $18.76.

It was a revelation to see the eagerness with which the committee members handled, and counted the money. Several of the less inhibited of their classmates asked permission to count the money, also. When this was allowed, it was evident that each child in the room experienced a craving to have a share in the actual handling of the funds. But there was not time enough before the breakfast date.

Retaining the Money for Further Use

It was arranged that a check would be used in payment for the breakfast expenses. Thus the actual money was retained in the classroom. During the ensuing days, each child had an opportunity to count the money.

Some difficulties were met. For example: a child might count by twenty-fives to fifty but be unable to add another twenty-five. He might need to build new concepts by comparing two dimes and a nickel with a quarter. He could be led to see that adding another quarter was like adding the three smaller pieces of money.

More Money Brought into the Classroom

To give further and more varied experience in counting money, the amount available was increased to thirty dollars. This amount was unevenly divided, each morning, and placed on five tables. For example:

Table	*Amount*
1	$ 6.24
2	$ 5.59
3	$ 5.88
4	$ 7.07
5	$ 5.22
Total	$30.00

The class assembled in five groups, around the tables. Each group counted their money; more than once, if necessary. When they were agreed on the amount, they checked with the teacher. When two groups had counted accurately, they exchanged places and counted more money.

Problem Solving

Through the means just described, facility in counting had increased. At this point, children went a step further. After counting the money on a given table, they were given a set of examples based upon their supply of money. Some such examples as the following were used: You now have $6.24.

1. If you spent $4.83, how much money would you have left?
2. If each of four friends gave you $1.07 more, how much money would you have then?
3. If you found two quarters and a dime, what would be your total amount then?
4. If three people in your group each spent 14 cents, how much money would you have left?
5. If you had four times as much money, how much would you have?
6. If your money were changed to pennies, how many pennies would you have?
7. If you divided the money between the people in your group, how much would there be for each? Would there be any pennies left over?

Sometimes children of a group made up original examples and computed the answers.

Playing Store

At the children's suggestion, a store was built. The usual array of grocery boxes and cans developed. Also, there were toys, jewelry, books and works of children's art. Price tags were made and displayed. A cash box was made available. Each child had several opportunities to play both customer and clerk. Frequently, the children came to the teacher in a double row: one was given a bill, the other a handful of change. The one with a bill bought from the other, who played clerk.

On one occasion the third grade was invited in. As they entered the room, each child was handed a dollar bill. Third graders selected articles, and fourth grade children were the clerks who sold to them. Both clerks and customers enjoyed this experience.

After this occasion the cash box was three dollars short. The fourth grade children were concerned. It was suggested that someone might have made a mistake and put some in his pocket. The boy who found that he had done so seemed as surprised as possible. Money had become rather commonplace in the room.

The Culminating Activity

A community leader, manager of a local department store, was invited to visit school and talk to the children. At his suggestion, a venture to his store was arranged instead.

It was proposed that each child be allowed to purchase an article, and also to sell something in the store. This would involve much preliminary planning. But the suggestion was appreciated as an opportunity for the children to have a unique experience. Thirty children were virtually to take over the store for a morning.

For preliminary planning, the manager and the classroom supervisor together toured the store. They listed the purchases to be made, together with the prices and the amounts of money to be supplied for each purchase. For example:

Article	*Price*	*Amount*
Baby carriage	$19.75	$20.00
2 Pyrex Pie Plates @ .45	.90	1.00
Lunch cloth	2.98	5.00

The total amount needed was calculated. A day was set for the invasion of the store. In the intervening days the children learned the use of sales slips. The manager trusted to the care of the classroom supervisor a check for $124.50, the total amount needed. He also designated a place where wraps could be conveniently laid in the morning. He requested (1) that children allow clerks to handle the cash carriages, and (2) that "purchased" articles be placed on a designated table.

Many details were still to be arranged. Individual envelopes were provided for the return of change.

Individual cards were written, with the help of student teachers, designating the purchases to be made. Two cards were required for each sale. They were worded as follows:

> Joan Bush
> Sell
> Women's Gown $2.98
> to Dotty Jean May

> Dotty Jean May
> Buy
> Women's Gown $2.98
> from Joan Bush

The Store Experience

Children were expectant and eager on the appointed morning. Three student teachers, who had helped with the planning, accompanied the class group.

The first stop was at the bank. Here a boy, of the children's selection, was allowed to cash the check. A bank officer suggested a tour of the bank. He personally conducted the group to the bookkeeper's department, and to the vault, where large packages of paper currency were seen. From the bank, the children went directly to the store.

The manager greeted the group cordially and then left them to carry out their plans freely.

The children removed their wraps. Then each child was given a card, with directions to make either a sale or a purchase. In addition, the purchasers were given the needed amount of money.

The children found the designated articles by scouting around, and the sales were made. Regular sales slips were used. The store clerks sent the money up in the carriages, and the change was returned to the child playing clerk, and counted into the hand of the purchaser.

One child who received the wrong change from the cashier noted it instantly, and had it returned for correction. This "error" had been planned as a challenge.

Finally the change was placed in an envelope. On the envelope was recorded the name of the child who made the purchase, the original amount received, the cost of the purchase, and the amount of change returned. Student teachers checked these accounts and retained the envelopes temporarily.

After every child had made either a purchase or a sale, cards were again distributed. Each child who had already played clerk, now made a purchase from one who had previously bought something. When the second round of buying was over, the cash envelopes were returned to the office. The cashier later reported that all funds

used by the children were accounted for. No money had been lost throughout the entire series of activities.

It may be of interest that at no time had mention of honesty been made to the class. It was assumed that the money would be used for the purposes for which it was provided. Trust in the integrity of the children was implied, but never spoken.

Outcomes

No formal arithmetic drill was given during the weeks in which these activities took place. The Stanford Achievement Test was given during the initial activities and again after the visit to the store. About five weeks had elapsed. The average scores showed normal progress, or better, in both computation and problem solving.

Social relationships had been strengthened through (1) group work in problem solving, (2) sharing responsibility in caring for money, and (3) buying and selling from each other.

A feeling of adequacy in handling funds had been developed.

A variety of concepts concerning money had been developed. *How much money?* could be conceived of as having specific meaning. Answers could be thought of in various combinations of coins, and amounts had come to connote something more than symbols graphically represented in books, or on paper.

Implications

It is not the intent of the author to suggest that all arithmetic be taught through activities or incidentally. On the contrary, she agrees with Morton[1] that incidental learning does not produce satisfactory results, and that there must be a systematic attack upon arithmetic.

Sueltz[2] warns of three weaknesses in the type of school that attempts to teach arithmetic incidentally through activity:

1. Important mathematical situations may be missed because they and their implications are not familiar to teacher and pupils.

2. Situations and problems may be investigated and mastery may be attempted without regard to sensible mathematical sequence.

3. Sufficient study and mastery of mathematical processes is not provided for through the investigation of such a topic as transportation in all its aspects.

It is true, however, that in arithmetic, as in any other subject, children often reveal a need of enrichment of experience background. Too often, through pressure of time, or through lack of insight and initiative a meager vicarious experience, or none at all, is provided.

No teacher will be likely to use just such a combination of activities as are reported in this article. Each teacher's cue comes from the vagueness of concepts revealed by the children under her tutorage.

It is desirable to teach some arithmetic through activity when:

1. to omit the activity will result in doing arithmetic mechanically without an understanding of application to everyday living.
2. the activity will clarify concepts and motivate interest.
3. the computation involved is at a point where further practice rather than initial teaching is needed.
4. the activity will provide sufficient practice to adequately increase the mathematical skill involved.

[1] Morton, Robert Lee. *Teaching Arithmetic in the Elementary School, Volume II, Intermediate Grades.*

[2] Sueltz, Ben A. "Curriculum Problems—Grade Placement." *National Council of Teachers of Mathematics, Sixteenth Yearbook, Arithmetic in General Education.* New York, Bureau of Publications, Teachers College, Columbia University, 1941, p. 125.

EDITOR'S NOTE: It should be noted in Dr. Linda Smith's experiment with money that most of the activities were carefully planned. The teacher knew what she wanted the children to do and the hoped-for outcomes in terms of attitudes, information, procedures, and processes. The experiment was organized because of a recognized need with this group of children. Dr. Smith is a specialist in reading and hence more conscious of the problems of concept building than many teachers might be. The use of real money is of course better than using "Play money."

Teaching Arithmetic with Calculators

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WHEN YOU WERE A YOUNGSTER in school, would you have given up a play period for a chance to stay indoors and do arithmetic?

Chances are you wouldn't, not unless you were extra fond of addition and subtraction. Arithmetic is not a subject which most youngsters choose in preference to baseball or jump-rope, yet in Short Hills, New Jersey the third graders did just that. The reason? They were learning arithmetic a new way—with calculating machines.

At the Glenwood Public School in Short Hills, third graders operated modern calculating machines during ten weeks of the school year to sharpen their skill with one of the three R's—arithmetic.

The idea of using calculating machines to teach arithmetic was first tried by the Monroe Calculating Machine Company in experiments at New York's Hunter College Elementary School. The educational principle behind the idea is simple: "Get a child to like a subject and he's bound to do better at it." The calculator is the interest bait. It does not, of course, eliminate any brain work on the part of the children. Far from it. The child must first learn to do his arithmetic on paper; he then uses the calculator to check his answers.

Just how well this theory works was illustrated by my third graders at the Glenwood School. They used the machines every day for ten weeks during the arithmetic period and would get very excited when I'd tell them we were going to do some arithmetic. In fact, one beautiful spring day I promised them a free play period, but we started using the calculators and they forgot all about playing. No one even mentioned it.

The youngsters took great pride in getting the right answers. They didn't rush through their paper work because they were competing with the machine and wanted their answers to be just as correct as the calculator's.

The Glenwood art teacher, who happened to look in on the class during the arithmetic period, remarked, "I never saw children so engrossed in what they were doing. No one was paying any attention to anyone else, they were so busy working." Then there was the downcast lad who had to drop out of the third grade in another section of town because his family was moving to the Glenwood School district. Unhappy because he had to leave his pals behind, his face lit up and his spirits brightened when he was told he'd be going into the class at Glenwood that used the calculators. One mother asked, "What's it all about? My child is so enthusiastic about arithmetic!" Enthusiasm is the biggest key. You must get a child enthusiastic about something before he'll learn.

Significantly, the machine used is one known as the Monroe "Educator"—a non-electric, hand-operated machine that's very simple for children to operate. Push-

ing buttons and turning handles puts glamour into a dull routine of adding, subtracting, multiplying and dividing. Perhaps even more important, the machine demonstrates the basic principles of arithmetic. The child learns from the machine that there are only two processes—addition and subtraction, and that all variations are merely a series of one of these.

States H. Richard Conover, principal of the Glenwood School, "The calculators are another visual teaching aid, so valuable in education. Up until the past few years arithmetic was one of the least interesting subjects to children. The machines are one of the improvements which have been made in teaching the subject. I was amazed at the enthusiasm and interest shown by the children in arithmetic. And if we can interest them in a subject, they have a better chance of learning."

Pointing out the value of competition, Mr. Conover comments that, "The calculators offer a challenge to pupils to do their best work so they won't be beaten by the machine. Public schools today don't place major emphasis on competition, but this kind of wholesome rivalry between pupil and machine is valuable."

In going through the mechanical operations of running the machine, the children learned certain arithmetical rules. For instance, pupils usually have trouble learning to place numbers in the correct columns. After using the calculators, however, they saw that each number had to line up in the proper column or their answers just wouldn't come out right. The machines were especially good for teaching multiplication facts and division. Using the calculator, too, is healthful for the pupil with less ability in arithmetic. He gains confidence because he can check his answers and find his own mistakes, thereby eliminating the fear of error that makes arithmetic so hateful to so many youngsters.

Another technique which has proved very effective in developing self assurance in pupils is the demonstration board. This consists of a large picture of the Educator, which is hung on the blackboard for all to see. At the start of each arithmetic period, one pupil is selected to explain to the class the various parts of the machine and to show how it must be manipulated for adding, subtracting, multiplying or dividing. It is amazing to see how eager the children are to get up and demonstrate the calculator to their classmates.

At Glenwood there were 28 pupils in my class. At first we tried sharing the "Educators," but we later found that having a machine for each child was much more effective. We used the calculators approximately 20 minutes of the arithmetic period, which lasted a maximum length of 50 minutes.

When the ten weeks were over, the third graders wrote 28 letters of thanks to the Monroe Company for the use of the machines. They ran something like this: "Thanks a lot for the 'Educators.' We liked them very much." "They helped us do our facts." "I know mine better now." "We loved the Monroe 'Educators' very much." "We learned to add, divide, multiply and subtract."

The enthusiasm these third graders developed for arithmetic is something I'll not forget. Arithmetic had suddenly taken on a new look for them. I'm sure, too, that for any children who use the calculators, arithmetic will be brighter for a long time to come.

EDITOR'S NOTE: This article points out how a teacher can use a computing machine to stimulate interest in learning certain types of arithmetic. Certainly the novelty of a machine and the challenge to meet its standards mentally should whet pupils. Just what phases of arithmetic instruction are helped by a machine? What phases are not suitable or perhaps hampered with the introduction of a computer? Should we, at this stage in our civilization, cease to teach addition and other processes except by machine? Will computers facilitate children to understand the mathematics of their environment and to think and draw conclusions? Teachers are asking about the proper role of computing machinery in the public school program. Who will explain what we might expect from a mechanical device and when and how to use it and what we cannot obtain through the use of a machine?

Codes for Boys and Girls

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In the November 1952 issue of *The Mathematics Teacher* Andree[1] suggested a unit on cryptography as a motivating factor in the teaching of General Mathematics. As a direct result of this article an in-service class in cryptography was taught by the author during the fall semester of 1953–1954. The class consisted of elementary school teachers, high school mathematics teachers and an elementary school principal.

The first half of the course was devoted to (1) a brief history of cryptography, (2) an explanation of the difference between a cipher and a code, and (3) cryptanalysis of the substitution cipher, both formal and informal.

The substitution cipher was used as the basis of the cryptanalysis because it is the cipher which can be used most successfully as a motivation factor in elementary mathematics.

The last half of the course was devoted to a discussion as to how cryptography could be used in teaching elementary mathematics. Each teacher[2] planned an instructional unit usable with his particular group of students. He then tried out his unit in his group and reported the results to the class.

A typical unit was the one planned by the elementary principal. He stated that when one of his teachers is excused for half-day professional meetings or visiting days, he takes the teacher's class. It was on such an occasion that he presented his cryptography unit to a fifth grade class in a Kirkwood, Missouri school. The procedure was as follows.

The following informal substitution cipher was put on the blackboard before the class assembled.

MVT NOGDCFNL TQSV UVVF
LVFM MG BGFNGF DVVM PL
MTUKV GF DGFNQY.[3]

Interest was thus aroused.

First the difference between a code and a cipher was explained to the class. The class was told that the cipher on the board was from a note that had been found in the pocket of a robber who had been shot down attempting to rob a jewelry store in New York City. Work in deciphering began by giving hints as to the importance of finding a key word and its possible location. After one or two false starts, the class quickly discovered the word "diamonds," and with interest stimulated by success, worked out the message.

Further explanation was made that any cipher can be broken. Deciphering is sometimes very complex, but with sufficient time and some luck can be accomplished. The class recognized that the amount of time required for the solution was important where immediate knowledge of the message was necessary, as in wartime.

A second cipher was given to the class in mimeographed form with one key phrase translated. This sheet was then put aside to be taken home and worked out at leisure.

Next it was discussed, if no clue or keyword had been given, how the problem might have been attacked. The question of letter frequency was fairly obvious to the class and they tried to determine which letters are most frequently used in English. One child was sent to the office to examine

[1] Andree, R. V., "Cryptography as a Branch of Mathematics." *The Mathematics Teacher*, Vol. 45, No. 7, pp. 503–509.

[2] Those teachers who taught in grades below the fifth wrote a unit usable in a higher grade.

[3] Andree, R. V., *op. cit.*, pp. 505.

the typewriter to see if he could determine the most frequently used letters on the keyboard. The chrome finish on the edges of the "t" and "e" was observed to have been so worn that the brass underneath showed. This gave a clue for frequency. Together the class tried to work out a frequency table of letters. It is surprising how close to the real frequency charts they came with this first attempt. They were then given a copy of the frequency tables for use of each single letter, double letters, initial letters, and final letters. The method of making a frequency table for initial and final letters as used in the given cipher was demonstrated on the board.

After this introduction, the following cipher was placed on the board and the class worked it out together.

THBIP SYAQ QIB MHB THBIP SYIQI SPIIQ FAISY SYI OJBX JD QJFJGJH KIMQI YI'Q HJS AH SYAQ YJFI OTS JHFX YAQ KJB YI QYIFFIB JTS YAQ GJTF MHB VIHS TK SJ YAQ LJB.

Success was achieved easily and interest in the procedure remained high for all the students throughout the lesson. At the end of the period another mimeographed cipher was given each pupil to be taken home and deciphered, making two to be done individually. That interest was high was evidenced by the busy groups of boys and girls who could be seen working together after school and the many correct solutions that were brought back with pride the next morning.

The apparent values from a unit in cryptography in the elementary classroom are:

1) Cryptography is an excellent model for deductive reasoning. Each time a cipher is broken, data must be collected and organized and hypothesis drawn from these data. Deductive reasoning is then used to test the validity of the hypothesis and to draw conclusions.

2) Mathematics and the language arts may be integrated.

3) A mathematical and social phase of arithmetic are brought together.

4) This topic is both educational and recreational. It is a worthy use of leisure time.

5) It is an excellent mode of getting children to work together. Cooperation of two or more pupils usually yields results more quickly.

6) Cipher work is a challenge. It has the element of mystery and the lure of the unknown. It must be completed to be satisfying.

7) Analysis and study of this type may be very simple or highly complex and thus may be suited to varying levels of ability.

EDITOR'S NOTE: The puzzle element of cipher analysis appeals to many pupils. The amount of arithmetic in simple codes may be very small and yet the stimulation and satisfaction which children receive may have far reaching results. This may be a good topic to use as an "extra" for bright pupils. However, certain pupils may go so far as to embarrass a teacher by their own inventions.

The Boston Meeting

The annual meeting of the National Council of Teachers of Mathematics will be held in Boston April 13–16. Headquarters will be in the Statler Hotel. Dr. Jackson Adkins of Phillips Exeter Academy is chairman of the local committee on arrangements. As usual at this annual meeting there will be a number of programs devoted to arithmetic and related topics. A more detailed program will be printed in the April issue of THE ARITHMETIC TEACHER.

More Rationalizing Division of Fractions

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The December, 1954 issue contained an article entitled "Rationalizing the Division of Fractions" by my colleague, Prof. Sam Duker. In the Editor's Note, following the article, readers are invited to indicate their reaction and to submit better methods.

Following is an explanation of another method. I do not know whether it is better, but it seems to be simpler in that it involves only *two* of the three understandings listed by Prof. Duker. These two are his first and third—"an understanding of the process of multiplications of fractions," and "an understanding that in any division process the quotient is not changed by multiplying both the dividend and divisor by the same quantity."

Let me illustrate the method I have in mind.

Example I.

$$3/4 \div 2/5$$

Step 1: Get the common denominator —20.

Step 2: Multiply dividend and divisor by the common denominator.

$$\begin{gathered} 20 \times 3/4 \div 20 \times 2/5 \\ 5 \times 3/1 \div 4 \times 2/1 \\ 15 \div 8 \end{gathered}$$

Step 3: Carry out the division, mentally or in written form:

$$\begin{array}{l} \quad 1\ 7/8 \\ 8/\overline{15} \end{array} \qquad \text{or } 15/8 = 1\ 7/8$$

Example II.

$$\begin{gathered} 4 \div 2/5 \\ 4/1 \div 2/5 \\ 5 \times 4/1 \div 5 \times 2/5 \\ 20 \div 10 = 20/10 = 2 \end{gathered}$$

Example III.

$$\begin{gathered} 2/3 \div 3/4 \\ 12 \times 2/3 \div 12 \times 3/4 \\ 4 \times 2/1 \div 3 \times 3/1 \\ 8 \div 9 = 8/9 \end{gathered}$$

The rule for the learner would be:

To divide by a fraction, first multiply the dividend and divisor by the common denominator of the fractions and then do the division.

The first step really has two parts—multiplying by the common denominator and then making the dividend and divisor each integers. But all this is implicit in the understanding of multiplication of fractions.

Another point to note is the gradation of the three examples. When the process is first introduced Example I should be used. When the pupils have gained some experience with it, the second is to be taught, and then the third. For the third, the teacher would have to show that the division of two integers can be shown as a fraction, viz: $8 \div 9 = 8/9$. Incidentally, some teachers, when they first teach the meanings of common fraction, include this one too, namely that a common fraction is an indicated division.

A comment is in order on the understanding of the concept of the reciprocal. I do not wish to convey the impression that this understanding is valueless. On the contrary, I feel that it has much value in developing a deeper insight into the nature of fractions. It is also a useful concept in high school mathematics. However, it is not essential to the process under consideration. Moreover, its use in the rationalizing of the division by a fraction is not so simple to explain even to prospective teachers. The number *1* is an elusive integer. When the divisor has been transmuted to

1, which makes the quotient equal the divident, students appear puzzled. Also, that which started out as a division exercise ends up a multiplication exercise.

The rule, therefore, as given by Prof. Duker—"*in order to divide by a fraction we multiply the dividend by the reciprocal of the divisor*" contains a difficult, puzzling idea for learners. And as for the old "*invert the divisor and multiply*" it is so cryptic a short cut, and uses such unarithmetical terminology (invert), that its value is open to serious doubt.

The method that I am suggesting has, it seems to me, these three merits: a) It reduces the number of understandings necessary for the process; b) it can be rationalized more readily, and c) it ends as a division exercise. As for the common denominator, pupils will have already acquired the skill in computing it in their work with the addition and subtraction of fractions.

Finally, may I add that the process in question is by no means an easy one, so that at best what we should try to get is as simple a method as possible, and one that would present the least difficulties in the rationalization.

EDITOR'S NOTE: Yes, the process of rationalizing division by a fraction is difficult for young children. This is particularly true when the process is viewed as a "science of numbers." Let us have the steps which teachers in grades five and six use. What is the role of objective materials and devices such as "cake-pan fractions" in this process?